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A versatile double Langmuir probe technique has been developed by incorporating analytical fits to Laframboise's numerical results for ion current collection by biased electrodes of various sizes relative to the local electron Debye length. Application of these fits to the double probe circuit has produced a set of coupled equations that express the potential of each electrode relative to the plasma potential as well as the resulting probe current as a function of applied probe voltage. These equations can be readily solved via standard numerical techniques in order to infer electron temperature and plasma density from experimental data. Because this method self-consistently accounts for the effects of sheath expansion, it can be readily applied to low-temperature plasmas with a wide range of densities without a priori tailoring of probe dimensions to the expected electron Debye length. The presented approach has been successfully applied to experimental measurements obtained in the plume of a low-power Hall thruster.

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Improved analysis techniques for cylindrical and spherical double probes

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A versatile double Langmuir probe technique has been developed by incorporating analytical fits to Laframboise's numerical results for ion current collection by biased electrodes of various sizes relative to the local electron Debye length. Application of these fits to the double probe circuit has produced a set of coupled equations that express the potential of each electrode relative to the plasma potential as well as the resulting probe current as a function of applied probe voltage. These equations can be readily solved via standard numerical techniques in order to infer electron temperature and plasma density from experimental data. Because this method self-consistently accounts for the effects of sheath expansion, it can be readily applied to low-temperature plasmas with a wide range of densities without a priori tailoring of probe dimensions to the expected electron Debye length. The presented approach has been successfully applied to experimental measurements obtained in the plume of a low-power Hall thruster.

I. INTRODUCTION

Langmuir probes of various geometries are widely used in the diagnosis of laboratory plasmas due in part to the straightforward nature of their experimental implementation. Of particular interest for many applications is the double probe, in which a variable voltage is applied between two electrodes and the resulting current characteristic is assessed in order to infer the local electron temperature and plasma density.[1,2] Double probes have an advantage over their single probe counterparts in that their mean potential floats with the plasma and therefore a stable reference electrode is not required. In addition, the current conducted through the double probe circuit is limited to the ion saturation current, as opposed to the much larger electron saturation current that can be collected by single probes.[1,2] The result is a less significant disturbance to the ambient plasma and reduced heating of probe electrodes.

A challenge that is common to all geometries of Langmuir probes is the fact that their interpretation requires knowledge of the relation between the ion current collected by a biased

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electrode and the local plasma parameters. Often interpretation of the probe characteristic is accomplished by assuming that the probe operates in one of two asymptotic regimes: the thin-sheath limit or the orbital motion limit (OML). In the thin sheath analysis, which is appropriate for very high plasma densities measured with relatively large probes, the plasma sheath is taken to be vanishingly thin relative to the probe dimensions such that all ions that enter into the sheath are ultimately collected by the probe. In the OML analysis, which is appropriate for low plasma densities and small probes, the plasma sheath is taken to be infinitely large compared to the probe dimensions such that the sheath does not limit the penetration of the electric field into the bulk plasma. In this case, the probe collects all ions whose momentum relative to the probe surface is insufficient to escape the electric field around an electrode that is biased negative with respect to the local plasma potential. While both the thin-sheath and OML analysis techniques have found widespread use, there are many practical cases in which laboratory plasma parameters fall between their respective regions of applicability. In these cases, a method is needed that appropriately accounts for the finite, but non-negligible extent of the plasma sheath.

One of the most extensive assessments of ion collection by a biased cylindrical or spherical electrode in a collisionless plasma was conducted by Laframboise, whose results were presented in graphical and tabular form for a range of probe radius to Debye length ratios (r_p/λ_D).[3] Subsequently multiple authors developed analytical fits to Laframboise's results, which were generally valid over a limited range of plasma parameters.[4,5,6] Recently, Steinbruchel and colleagues developed an analytical parameterization that is applicable over a wide range of r_p/λ_D for cold ions ($T_i/T_e<<1$), and they successfully applied those results to single Langmuir probes of both cylindrical and spherical geometry.[7,8,9] In the present work, we extend those results to the double probe geometry in a self-consistent manner that, unlike the single probe, does not require knowledge of the local plasma potential in order to determine electron temperature and plasma density.

II. PROBE THEORY

A typical floating double probe circuit is shown in Fig. 1, where the notation I_{+n}, I_{en} depicts the positive sense of the ion and electron current, respectively, collected by electrode n. Defining V_n as the voltage of electrode n with respect to the local plasma potential and applying Kirchoff's laws leads to Eqns. 1 and 2. Current continuity through the circuit ensures that the magnitude of the probe current, I_p, can never exceed the maximum current that can be collected by a single electrode, i.e. the ion saturation current. For this reason, as well as the fact that the electron saturation current is always much larger than the ion saturation current, both electrodes are electron repelling (i.e. $V_1<0$, $V_2<0$), with one electrode biased slightly above the plasma floating potential and the other below it.[10] This allows the electron currents to be written as in Eqn. 3, where A is the surface area of a single electrode, e is the magnitude of the electron charge, n_0 is the local number density of the undisturbed plasma, m_e is the electron mass, ξ is the local electron temperature in electron volts (i.e. $k_B T_e/e$), and I_{e0} is the thermal electron current to a probe at plasma potential.[7] It is assumed that the undisturbed plasma is both quasineutral and singly-ionized such that the electron and ion densities are equal. Manipulation of Eqns. 1-3 results in the fundamental double probe characteristic given by Eqn. 4.

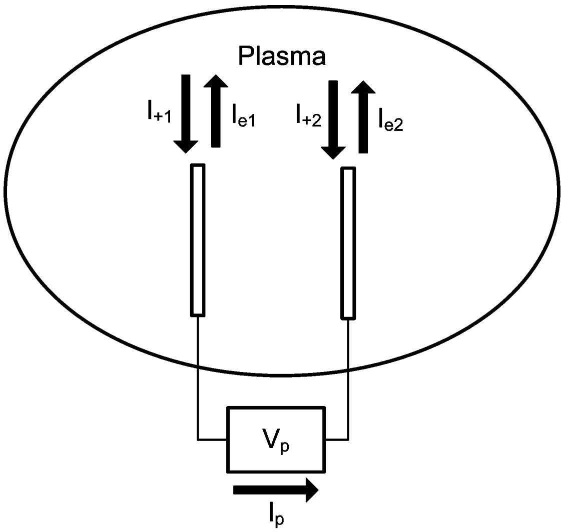

Figure 1: A double probe circuit depicting the positive sense of the ion and electron current to each electrode.

$$I_P = I_{+1} - I_{e1} = I_{e2} - I_{+2} \tag{1}$$

$$V_P = V_2 - V_1 \tag{2}$$

$$I_{en} = A_n e^{3/2} n_0 \left(\frac{\xi}{2\pi m_e} \right)^{1/2} \exp\left(\frac{V_n}{\xi} \right) = I_{e0} \exp\left(\frac{V_n}{\xi} \right) \tag{3}$$

$$I_P = I_{+1} \tanh\left(\frac{V_P}{2\xi} \right) + \frac{I_{+1} - I_{+2}}{\exp\left(\frac{V_P}{\xi} \right) + 1} \tag{4}$$

It is worth noting that the derivation of the above double probe characteristic makes no assumptions about the features of the ion current collection mechanism. Equation 4 is valid regardless of r_P/λ_D so long as the electron distribution function is Maxwellian such that Eqn. 3 applies. In the idealized case where the ion saturation current is independent of the applied bias potential (i.e., $I_{+1}=I_{+2}$), the characteristic above reduces to the original symmetric double probe formulation of Johnson and Malter.[11]

It has been shown that the ion current collected by a spherical or cylindrical probe can be represented by Eqn. 5, where I_0 is the ion current at the sheath edge given by Eqn. 6.[7,9] The fit parameters a and b are functions of r_P/λ_D and are given by the expressions in Table I, which have been reported to produce correlation coefficients greater than 0.997 over the range $3<r_P/\lambda_D<50$.[9] For the cylindrical probe at values of r_P/λ_D less than 3, the true b parameter deviates from the value given by Table I and can be approximated as having a constant value of 0.5 over the range $0< r_P/\lambda_D<3$.[8]

$$I_{+n} = I_0 a \left(\frac{-V_n}{\xi} \right)^b \tag{5}$$

$$I_0 = e^{3/2} n_0 A \left(\frac{\xi}{2\pi m_i} \right)^{1/2} \tag{6}$$

Table I. The fit parameters a and b for $3<r_P/\lambda_D<50$ (from Ref. 9).

Probe geometry	a	b
cylindrical	$1.18-0.00080(r_P/\lambda_D)^{1.35}$	$0.0684+(0.722+0.928 \times r_p/\lambda_D)^{-0.729}$
spherical	$1.58+(-0.056+0.816 \times r_P/\lambda_D)^{-0.744}$	$-0.933+(0.0148+0.119 \times r_P/\lambda_D)^{-0.125}$

The expression above for the collected ion current can be inserted into Eqn. 4 to yield the final double probe current characteristic, which is given by Eqn. 7. To make use of this expression in deducing plasma parameters from experimental data, one must first relate the potential of electrode 1, V_1, to a directly measurable quantity such as V_P. This is accomplished by noting that the probe as a whole floats such that no net current is drawn from the plasma, as shown by Eqn. 8. This equation can be solved implicitly to yield V_1 as a function of V_P for a given ion species, electron temperature, and predetermined values of the fit parameters a and b.

$$\frac{I_P}{I_0} = a\left(\frac{-V_1}{\xi}\right)^b \tanh\left(\frac{V_P}{2\xi}\right) + \frac{a\left(\frac{-V_1}{\xi}\right)^b - a\left(\frac{-(V_1+V_P)}{\xi}\right)^b}{\exp\left(\frac{V_P}{\xi}\right)+1} \tag{7}$$

$$I_{+1} + I_{+2} - I_{e1} - I_{e2} = a\left[\left(\frac{-V_1}{\xi}\right)^b + \left(\frac{-(V_1+V_P)}{\xi}\right)^b\right] - \left(\frac{m_i}{m_e}\right)^{1/2} \exp\left(\frac{V_1}{\xi}\right)\left[1+\exp\left(\frac{V_P}{\xi}\right)\right] = 0 \tag{8}$$

Double probe current characteristics resulting from Eqns. 7 and 8 are shown in Fig. 2 for cylindrical probes and a range of r_P/λ_D. The dashed line in Fig. 2 shows the double probe characteristic that results when one assumes the thin-sheath ion collection mechanism originally described by Bohm.[1] As shown in Fig. 2, for large r_P/λ_D the results given in Eqn. 7 are in very good agreement with the thin-sheath approximation. As r_P/λ_D decreases (i.e. for small probes and low plasma densities) the collected ion current becomes disproportionately large relative to I_0. Similar characteristics are shown in Fig. 3 for the case of spherical probe electrodes.

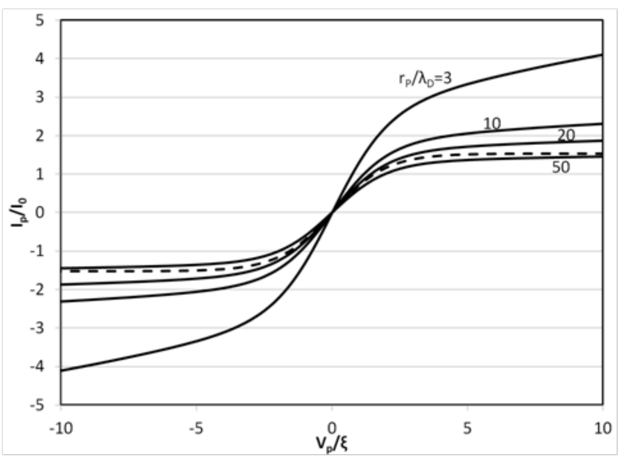

Figure 2: Double probe characteristics for various ratios of probe radius to Debye length for cylindrical electrodes. The characteristic associated with the thin-sheath approximation is shown as a dashed line.

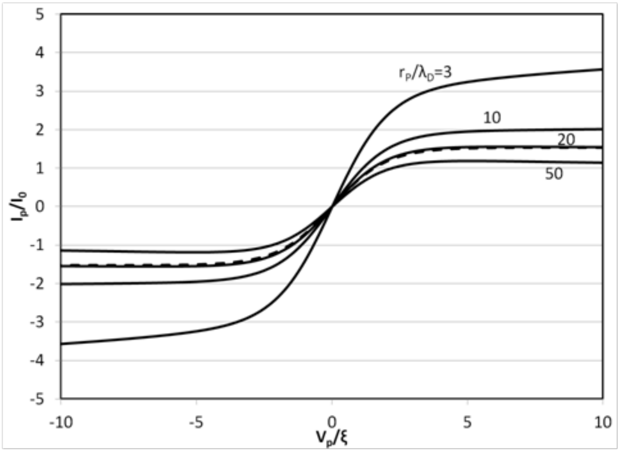

Figure 3: Double probe characteristics for spherical probes and various ratios of probe radius to Debye length. The dashed line represents the characteristic associated with the thin-sheath approximation.

III. SAMPLE EXPERIMENTAL RESULTS

With the relations established in Section II, a straightforward iterative approach may be used to determine plasma density and electron temperature from experimental I_P and V_P data. This approach starts with initial guesses for n_0 and ξ, which can be conveniently obtained using the typical thin-sheath approximation. These initial values are, in turn, used to calculate preliminary estimates of λ_D, a, and b. Standard numerical root finding techniques may then be applied to Eqn. 8 to obtain V_1 at each probe voltage, V_P. These values are then applied to Eqn. 7 and a numerical curve fitting routine, such as the Levenberg-Marquardt method available in many commercial software packages, is used to determine the values of n_0 (or I_0) and ξ that provide a best fit to the experimental data. These values of plasma density and electron temperature are then used to update λ_D, a, and b, and the procedure continues iteratively until it converges on a self-consistent set of values that satisfy Eqns. 7 and 8.

The method described above was utilized to measure plasma properties in the plume of a low-power Hall thruster.[12] Measurements were conducted in a spherical vacuum chamber with a diameter of 30-feet, which was evacuated to a background pressure of approximately $5\text{x}10^{-6}$ Torr. A double probe consisting of two cylindrical tungsten electrodes mounted in alumina tubes for insulation was placed at various locations within the thruster plume while a laboratory sourcemeter swept the probe voltage, V_P, and measured the resulting current, I_P. Each electrode was 4.8 mm in diameter and 196.9 mm long. The cylindrical probes were pointed at the exit of the Hall thruster such that the plasma flow was directed along the axis of each electrode. It has been shown that cylindrical probes oriented in this manner can be analyzed as if the plasma were at rest provided the probe is of sufficient length.[13] It is possible to incorporate the effect of directed ion flux to the probe end by adding an appropriate expression for this current to the expression for ion current in Eqn. 5 and propagating this addition through the subsequent equations. No such correction was made in the present work due to the large probe length-to-diameter ratio, which can be expected to make the end effect negligible.

Sample probe data obtained at different locations in the plume are presented in Fig. 4. Experimental data points are depicted as discrete symbols while the best fit satisfying Eqns. 7 and 8 is shown as a solid line. The represented data correspond to plasma densities of approximately 1.3×10^{17} m^{-3} and 1.2×10^{15} m^{-3}. The distinctly different shapes of the I_P versus V_P characteristics predicted for high and low values of r_P/λ_D can be seen in Figs. 4a and 4b where the probe radius to Debye length ratios are 8.3 and 0.3, respectively.

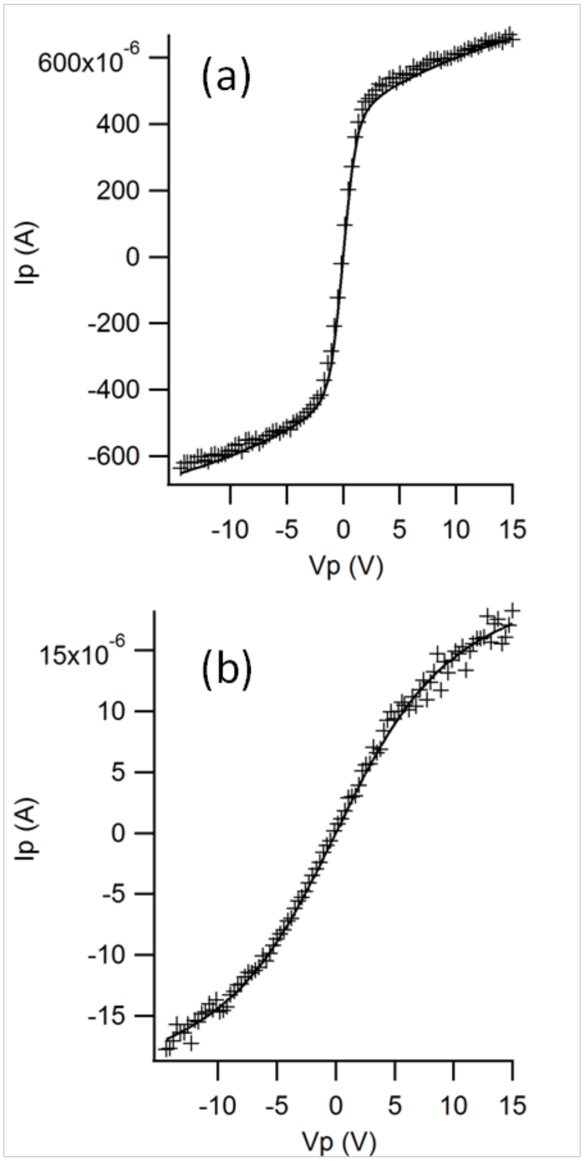

Figure 4: Sample double probe measurements taken at various locations in a plasma plume. Inferred plasma parameters are (a) n_0=1.3x10^{17} m^{-3}, ξ=0.56 eV, r_p/λ_D=8.3, and (b) n_0=1.2x10^{15} m^{-3}, ξ=3.92 eV, r_p/λ_D=0.3. The solid line in each graph is the best fit that satisfies Eqns. 7 and 8.

IV. CONCLUSIONS

A double probe method has been developed that incorporates analytical fits to Laframboise's numerical results for ion current collection by biased electrodes of various sizes relative to the electron Debye length. Propagation of these fits through the double probe circuit relations results in a set of coupled equations expressing the potential of each electrode relative

to the plasma potential and the resulting probe current. The resulting set of equations is well-suited to being solved via standard numerical techniques in order to infer the local electron temperature and plasma density. The presented approach has been successfully applied to experimental measurements obtained in the plume of a low-power Hall thruster.
Because this method self-consistently accounts for the effects of sheath expansion, it can be readily applied to low-temperature plasmas with a wide range of densities without a priori consideration of the relationship between probe dimensions and the electron Debye length.

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